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Technical Paper

Comparison of creep models and experimental verification of creep coefficients for normal and high strength concrete

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Abstract: A concrete structure when subjected to sustained load presents progressive strain over time, which is associated with the creep phenomenon. The creep characteristic of high strength concrete assumes importance in the back drop of increase in prestressed concrete constructions. The paper covers the comparison of creep coefficients with different creep models like Bazant's B-3, ACI, AASHTO, GL-2000 and FIB model code 2010 for concrete mixes having water to cementitious ratio of 0.47, 0.36, 0.27 and 0.20. The comparison of different models are done for a relative humidity of 60 percent and design life of 100 years. For comparison of creep coefficient using different models the age at loading are kept as 7, 28 and 365 days. Thereafter, values are compared with experimentally obtained results of concrete mixes having water to cementitious ratio of 0.47 and 0.20 for age at loading of 28 days and up to 180 days loading period. Time induced creep strain of high strength concrete is determined using creep rig of capacity 2000 kN. Creep strains are measured at regular time intervals on concrete designed with water to cementitious ratio of 0.47 and 0.20 wherein fly ash and silica fume were also used.

Keywords: Creep coefficient; normal strength concrete; high strength concrete; creep model.

1 Introduction

Creep performance is an important index in the long-term properties of concrete, and the linear compressive creep deformation can reach 1-4 times of the short-term elasticity compressive deformation. Therefore, the creep behaviour must be considered in the design of concrete structures in order to provide necessary safety and serviceability. For the important engineering structures, creep experiment of the specimen, which is made from the same concrete used in the structures, is the most reliable method. However, due to the complexity and diversity, there are not always sufficient condition to carry out creep experiment, so the empirical formula fitted from the obtained experimental data is essential [1]. There are many creep models available internationally, such as CEB-FIP series models, ACI 209 series models, GL-2000 model, AASHTO, B3 model, China Academy of Building Research model, Zhu Bofang model and Li Chengmu model et al. [2-7]. However, there are many differences in the influence factors, formula forms, applicable scope and prediction accuracy of these models due to limitation of specific experimental condition and the emphasis of different researchers. The correction factor of mixture ratio of concrete was given in CEB-FIP series models. The correction factor of collapsibility, sand ratio and air content were considered in ACI 209 series models. The correction factor of water cement ratio, cement content, sand ratio and concrete density was considered in B3 model. Recent research relates the creep response to the packing density distributions of calcium silicate-hydrates. At high stress levels, additional deformation occurs due to the breakdown of the bond between the cement paste and aggregate particles [8-15]. Therefore, designers and engineers need to know the creep properties of concrete and must be able to take them into account in the structure analysis. As per IS: 456-2000 [16], creep of concrete depends on the constituents of concrete, size of the member, environmental conditions (humidity and temperature), stress in the concrete, age at loading and the duration of loading. As long as the stress in concrete does not exceed one-third of its characteristic compressive strength, creep may be assumed to be proportional to the stress. High strength concrete is significantly in use now a days in number

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of concrete structures, the most common applications being the columns of high rise buildings, long span bridges, longer spans for beams or fewer beams for a given span length, offshore structures, etc. High-strength concrete is a more sensitive material than normal strength concrete and it must be treated with care both in design and in construction. The aim of the paper is to compare the creep coefficients with different creep models like Bazant's B-3, ACI, AASHTO, GL-2000 and FIB model code 2010 for concrete mixes having water to cementitious ratio of 0.47, 0.36, 0.27 and 0.20. The comparison of different models is done for a relative humidity of 60 percent and design life of 100 years. For comparison of creep coefficient using different models the age at loading are kept as 7, 28 and 365 days. Thereafter, values are compared with experimentally obtained results of concrete mixes having water to cementitious ratio of 0.47 and 0.20 for age at loading of 28 days.

2 Experimental program

2.1 Concrete ingredients:

Crushed aggregate with a maximum nominal size of 20 mm was used as coarse aggregate and natural riverbed sand confirming to Zone II as per IS: 383 was used as fine aggregate. Their physical properties are given in Table 1. The petrographic studies conducted on coarse aggregate indicated that the aggregate sample is medium grained with a crystalline texture and partially weathered sample of granite. The major mineral constituents were quartz, biotite, plagioclase-feldspar and orthoclase-feldspar. Accessory minerals are calcite, muscovite, tourmaline and iron oxide. The petrographic studies of fine aggregate indicated that the minerals present in order of abundance are quartz, orthoclase-feldspar, hornblende, biotite, muscovite, microcline-feldspar, garnet, plagioclase-feldspar, tourmaline, calcite and iron oxide. For both the coarse aggregate and fine aggregate sample the strained quartz percentage and their Undulatory Extinction Angle (UEA) are within permissible limits as per IS: 383-2016 (Strain Quartz percentage less than 20% and Undulatory Extinction Angle less than 15^o). The silt content in fine aggregate as per wet sieving method is 0.70 percent.

Ordinary Portland cement (OPC 53 Grade) with fly ash and silica fume are used in this study. The chemical and physical compositions of cement OPC 53 Grade, Properties of fly ash and silica fume are given in Table 2. Polycarboxylic group-based superplasticizer for w/c ratio 0.36, 0.27 and 0.20 and Naphthalene based for w/c ratio 0.47 complying with requirements of Indian Standard: 9103 is used throughout the investigation. Water complying with requirements of IS: 456-2000 for construction purpose was used. The 3 days, 7 days and 28 days' compressive strength of cement OPC 53 Grade were 36.00 MPa, 45.50 MPa and 57.50 MPa respectively. The 28 days' compressive strength of controlled sample and sample cast with fly ash was 38.53 MPa and 31.64 MPa respectively, when testing was done in accordance with IS: 1727. The 7 days' compressive strength of controlled sample and sample cast with silica fume was 12.76 MPa and 14.46 MPa respectively, when testing was done in accordance with IS: 1727.

2.2 Mix design details

In this study, the four different mixes with w/c ratio 0.47, 0.36, 0.27 and 0.20 using granite aggregate were selected for studying creep coefficient. The slump of the fresh concrete was kept in the range of 75-100 mm. A pre-study was carried out to determine the optimum superplasticizer dosage for achieving the desired workability based on the slump

Table 1 – Properties of aggregates

Property		Coarse Aggregate 20 mm	Coarse Aggregate 10 mm	Fine Aggregate
Specific gravity		2.83	2.83	2.64
Water absorption (%)		0.3	0.3	0.8
Sieve Analysis Cumulative Percentage Passing (%)	20mm	98	100	100
	10 mm	1	68	100
	4.75 mm	0	2	95
	2.36 mm	0	0	87
	1.18 mm	0	0	68
	600 µ	0	0	38
	300 µ	0	0	10
	150 µ	0	0	2
	Pan	0	0	0
Abrasion, Impact & Crushing Value		19, 13, 19	-	-
Flakiness % & Elongation %		29, 25	-	-

Table 2 – Physical, chemical and strength characteristics of cement

Characteristics		OPC -53 Grade	Silica Fume	Fly Ash
Physical Tests:				
Fineness (m^2/kg)		320.00	22000	403
Soundness Autoclave (%)		00.05	-	-
Soundness Le Chatelier (mm)		1.00	-	-
Setting Time Initial (min.) & (max.)		170.00 & 220.00	-	-
Specific gravity		3.16	2.24	2.2
Chemical Tests:				
Loss of Ignition (LOI) (%)		1.50	1.16	-
Silica (SiO_2) (%)		20.38	95.02	-
Iron Oxide (Fe_2O_3) (%)		3.96	0.80	-
Aluminum Oxide (Al_2O_3)		4.95	-	-
Calcium Oxide (CaO) (%)		60.73	-	-
Magnesium Oxide (MgO) (%)		4.78	-	-
Sulphate (SO_3) (%)		2.07	-	-
Alkalis (%)	Na_2O & K_2O	0.57 & 0.59	-	
Chloride (Cl) (%)		0.04	-	-
IR (%)		1.20	-	-
Moisture (%)		-	0.43	-

Table 3 – Concrete mix design details for study done

W/Cem	Total Cementitious Content [Cement + Fly ash + Silica Fume] (kg/m^3)	Water Content (kg/m^3)	Admixture % by weight of Cement	Fine Aggregate (kg/m^3)	Coarse Aggregate		28-Days Compressive strength	
					10 mm (kg/m^3)	20 mm (kg/m^3)	Cube (MPa)	Cylindrical (MPa)
0.47 (Mix-A)	**362** (290+72+0)	170	0.40	650	777	518	45.72	36.57
0.36 (Mix-B)	**417** (334+83+0)	150	0.35	726	730	487	68.57	57.14
0.27 (Mix-C)	**525** (400+75+50)	140	1.00	692	754	406	88.60	76.37
0.20 (Mix-D)	**750** (563+112+75)	150	1.16	536	640	427	103.55	90.83

cone test as per Indian Standard. The mix design details are given in Table 3. Adjustment was made in mixing water as a correction for aggregate water absorption. For conducting studies, the concrete mixes were prepared in pan type concrete mixer. Before use, the moulds were properly painted with mineral oil, casting was done in three different layers and each layer was compacted on vibration table to minimize air bubbles and voids. After 24 hours, the specimens were demoulded from their respective moulds. The laboratory conditions of temperature and relative humidity were monitored during the different ages at 27±2°C and relative humidity 65% or more. The specimens were taken out from the tank and allowed for surface drying and then tested in saturated surface dried condition.

3 Creep models

3.1 Creep as per B-3 model

This model (B3) was developed by Bazant and Baweja [5] and described by ACI in 1997. The B3 Model has been found to be useful for both simple and complex structures and it clearly separates basic and drying creep. As per B3 model, for constant stress applied at age at loading t', Total strain at time t,

$$\epsilon(\mathrm{t}) = J(t,t')\sigma + \epsilon_{sh}(t) + \alpha\Delta\mathrm{T}(\mathrm{t}) \qquad (1)$$

Where, J(t, t') is the compliance function = strain (creep plus elastic) at time t caused by a unit uniaxial constant stress applied at age t' in days', σ = uniaxial stress, ϵ = strain, ϵ_{sh} =shrinkage strain (negative if volume decreases), ΔT(t) = temperature change from reference temperature at time t, and α = thermal expansion coefficient.

The compliance function may further be decomposed as

$$J(t,t') = q1 + C_0(t,t') + C_d(t,t',t_0) \quad (2)$$

where, q1 = instantaneous strain due to unit stress, q1 = 0.6 x 10^6 /E_{28} and E_{28} (MPa) = $4734(f_c)^{0.5}$, C_0(t, t') = compliance function for basic creep (creep at constant moisture content and no moisture movement through the material), and C_d(t, t', t_0) = additional compliance function due to simultaneous drying.

The creep coefficient, φ(t, t′) should be calculated from the compliance function,

$$\varphi(t,t') = E(t')J(t,t') - 1 \quad (3)$$

where, E(t′) = (static) modulus of elasticity at loading age t ′

Calculations of Creep and Time Dependent Strain Components

The total basic creep compliance is obtained by equation as follows:

$$C_0(t,t') = q2 \cdot Q(t,t') + q3 \cdot \ln[1 + (t-t')^n] + q4 \cdot \ln(t/t') \quad (4)$$

where, q2, q3 and q4 represent the aging viscoelastic compliance, non-aging viscoelastic compliance, and flow compliance respectively, as deduced from the solidification theory, $q2 = 185.4\ c^{0.5}\ f_c^{-0.9}$, $q3 = 0.29(w/c)^4.q2$, $q4 = 20.3(a/c)^{-0.7}$.

The values of Q(t, t′) can be obtained from the following approximate formula (derived by Bazant and Prasannan, 1989 [17]) which has an error of less than 1% for n = 0.1 and m = 0.5;

$$Q(t,t') = Q_f(t')\left[1 + \left(\frac{Q_f(t')}{Z(t,t')}\right)^{r(t')}\right]^{-1/r(t')} \quad (5)$$

where, $r(t') = 1.7(t')^{0.12}+8$, $Z(t,t') = (t')^{-m}\ln[1+(t-t')^n]$ (m=0.5, n=0.1), $Q_f(t') = [0.086(t')^{2/9}+1.21(t')^{4/9}]^{-1}$

Additional Creep Due to Drying (Drying Creep)

$$C_d(t,t',t_0) = q5 \cdot \left[e^{-8H(t)} - e^{-8H(t_0')}\right]^{0.5} \quad (6)$$

If t ≥t'_0, t'_0= max(t′ , t_0). Otherwise, C_d(t, t′, t_0) = 0, t'_0 is the time at which drying and loading first act simultaneously, and

$$H(t) = 1 - (1-h)S(t) \quad (7)$$

where, $q5 = 7.57 \times 10^5 .f_C^{-1}. |\varepsilon_{sh\infty}|^{-0.60}$.

$$\epsilon_{sh\infty} = \epsilon_{s\infty}\left(\frac{E(607)}{E(t_0+\tau_{sh})}\right) \quad (8)$$

where,

$$E(t) = E(28)\left(\frac{t}{4+0.85t}\right)^{0.5} \quad (9)$$

$$\epsilon_{s\infty} = -\alpha 1\alpha 2(1.9\times 10^{-2} w^{2.1} f_c^{-0.28} + 270)\ \ (in\ \ 10^{-6}) \quad (10)$$

This means that $\varepsilon_{s\infty} = \varepsilon_{sh\infty}$ for t_0 = 7 days and τ_{sh} = 600 days.

Time dependence: $S(t) = \tanh((t-t_0)/\tau_{sh})^{0.5}$, size dependence: $\tau_{sh} = k_t(k_s.D)^2$, effective cross-section thickness (D = 2v/s) which coincides with the actual thickness in the case of a slab, v/s = volume to surface ratio of the concrete member. $k_t = 295740.59 \times t_0^{-0.08}.f_c^{-0.25}$ days/cm^2, k_s is the cross-section shape factor (Table 5).

High accuracy in this respect is not needed $k_s \approx 1$ can be assumed for analysis.

Following parameters and coefficients were considered while making calculations for experimental mixes using creep and shrinkage prediction model B3 by Zdenek P. Bazant and Sandeep Baweja,

- Type I cement was used in this study. Hence, α_1 was taken as 1.
- Since all the samples were sealed by wrapping in Butyl Rubber Sheet up to 28 days, α_2 was taken as 1.2
- Age at which drying of specimen began was taken as 28 days.
- Relative humidity of environment during curing and loading was maintained at 60% and same was used for calculations.
- Type of specimen was considered as infinite cylinder. Hence, k_S was taken as 1.15.
- All the other factors were calculated using above mentioned formulas by using different values of f_{cm}, t, t_0 and other parameters associated to individual mixes.

Table 4 – Coefficients based on cement type and curing conditions

α1	1.0	for type I cement
	0.85	for type II cement
	1.1	for type III cement
α2	0.75	for steam-curing
	1.2	for sealed or normal curing in air with initial protection against drying
	1.0	for curing in water or at 100% relative humidity.

Table 5 – Cross-section shape factor (k_s)

k_s	1	Infinite slab
	1.15	Infinite cylinder
	1.25	Infinite square prism
	1.30	Sphere
	1.55	Cube

3.2 Creep as per FIB model code 2010

The fib 2010 Model [3] was introduced by the International Federation for Structural Concrete in 2013. As per FIB model code 2010, within the range of service stresses $|\sigma_c| \leq 0\ 4.f_{cm}(t_o)$, creep is assumed to be linearly related to stress.

For a constant stress applied at time t_o this leads to creep strain at age of concrete t,

$$\epsilon_{cc}(t,t_0) = (\sigma_c(t_0)/E_{ci})\varphi(t,t_0) \qquad (11)$$

where, $\varphi(t, t_o)$ is creep coefficient, E_{ci} is the modulus of elasticity in MPa at the age of 28 days.

The stress dependent strain $\varepsilon_{c\sigma}(t, t_o)$,

$$\epsilon_{cc}(t,t_0) = \sigma_c(t_0)\left(\frac{1}{E_{ci}(t_0)} + \frac{\varphi(t,t_0)}{E_{ci}}\right) = \sigma_c(t_0)J(t,t_0) \qquad (12)$$

where, $J(t, t_o)$ is the creep function or creep compliance, representing the total stress dependent strain per unit stress and $E_{ci}(t_o)$ is the modulus of elasticity at the time of loading t_o.

Creep coefficient

The creep coefficient may be calculated from

$$\varphi(t,t_0) = \varphi_0\beta_c(t,t_0) \qquad (13)$$

where, φ_o is the notional creep coefficient and $\beta_c(t, t_o)$ is the coefficient to describe the development of creep with time after loading, t is the age of concrete in days at the moment considered and t_o is the age of concrete at loading in days.

$$\varphi_0 = \varphi_{RH}\beta(f_{cm})\beta(t_0) \qquad (14)$$

where, $\beta(f_{cm}) = 16.8/(f_{cm})^{0.5}$, $\beta(t_o) = 1/\ (0.1 + t_0^{0.2})$, and

$$\varphi_{RH} = \alpha_2\left[1 + \alpha_1\left(\frac{1-\frac{RH}{100}}{0.1h^{\frac{1}{3}}}\right)\right] \qquad (15)$$

f_{cm} is the mean compressive strength at the age of 28 days in MPa, RH is the relative humidity of the ambient environment in %. $h = 2A_c/u$ = notional size of member in [mm], where Ac is the cross-section in mm² and u is the perimeter of the member in contact with the atmosphere in mm. $\alpha_1 = (35/f_{cm})^{0.7}$ and $\alpha_2 = (35/f_{cm})^{0.2}$.

The development of creep with time, $\beta_c(t, t_o)$, is described by:

$$\beta_c(t,t_0) = \left[\frac{t-t_0}{\beta_H + t - t_0}\right]^{0.3} \qquad (16)$$

where

$$\beta_H = 1.5h[1 + (1.2RH/100)^{18}] + 250\alpha_3 \leq 1500\alpha_3 \qquad (17)$$

and $\alpha_3 = (35/f_{cm})^{0.5}$

Following parameters and coefficients were considered while making calculations for experimental mixes using FIB model code 2010,

- Relative humidity of environment during curing and loading was maintained at 60% and same was used for calculations
- All the samples were concrete cylinders having diameter 150 mm and height 300 mm
- All the other factors were calculated using above mentioned formulas by using different values of f_{cm} t, t_0 and other parameters associated to individual mixes.

3.3 Creep as per AASHTO 2014 model

The AASHTO Model [18] is described by AASHTO LRFD Bridge Design Specifications 7th Edition (Section 5.4.2.3) in 2014. The creep compliance $J(t, t_o)$ is given by,

$$J(t,t_0) = \frac{1}{E_{cm28}} + \frac{\varphi(t,t_0)}{E_{cm28}} \qquad (18)$$

where

$$E_{cm28}(\text{MPa}) = 0.043K_1\gamma^{1.5}(f_{cm28})^{0.5} \qquad (19)$$

K_1 = correction factor for source of aggregate to be taken as 1.0 unless determined by physical test. γ = concrete unit weight (kg/m³). Creep coefficient $\varphi(t, t_o) = 1.9.k_s.k_{hc}.k_f.k_{td}.t_o^{-0.118}$. Where, k_f = factor for the effect of concrete strength, $k_f = 35/(7+f_{cmto})$. k_s = factor for the effect of volume-surface ratio of the component, k_s= 1.45-0.0051(V/S), k_{hc}= 1.56 – 0.008H, where H is the relative humidity (%), $k_{td} = [t / (61 – 0.58f_{cmto} + t)]$.

Following parameters and coefficients were considered while making calculations for experimental mixes using AASHTO 2014 model

- Unit weight of concrete was considered as 2400 kg/m³.
- Since all the samples were cylindrical concrete specimen having diameter 150 mm and height 300 mm, V/S was taken as 0.03.
- Relative humidity of environment during curing and loading was maintained at 60% and same was used for calculations.
- K_1 was taken as 1 for all the mixes.
- All the other factors were calculated using above mentioned formulas by using different values of f_{cm}, t, t_0, and other parameters associated to individual mixes.

3.4 Creep as per ACI 209R-92 model

The American Concrete Institute recommends the ACI 209 Model [19] as the current standard code model. The creep compliance function $J(t, t_o)$ that represents the total stress-dependent strain by unit stress is given by

$$J(t,t_0) = \frac{1+\varphi(t,t_0)}{E_{cm}(t_0)} \quad (20)$$

where, $\varphi(t, t_o)$ is creep coefficient.

$E_{cm}(t_o)$ = modulus of elasticity at the age of loading (MPa) is given by $E_{cm}(t_o)$ in MPa $=(0.043)\gamma^{3/2}.(f_{cm}(t_o))^{0.5}$

γ is concrete unit weight in kg/m^3 and $f_{cm}(t_o)$ mean concrete compressive strength at age of loading. $f_{cm}(t_o) = f_{cm28}.[t_o/(a+b.t_o)]$, where f_{cm28} is the average 28-day concrete compressive strength (MPa) a and b are constants according to table 6 below.

Table 6 – a and b based on curing conditions

Type of cement	Moist cured concrete	Steam cured concrete
I	a = 4.0, b = 0.85	a = 1.0, b = 0.95
III	a = 2.30, b = 0.92	a = 0.70, b = 0.98

$$\varphi(t,t_0) = \varphi_u\left[\frac{(t-t_0)^{0.6}}{10+(t-t_0)^{0.6}}\right] \quad (21)$$

where, $\varphi_u = 2.35\ \gamma_H.\ \gamma_{to}.\ \gamma_s.\ \gamma_{vs}.\ \gamma_\alpha.\ \gamma_\psi$, t_o = age of concrete at loading (days), t = age of concrete (days), H= relative humidity (%), φ_u = ultimate creep coefficient.

Relative humidity correction factor, γ_H

$$\gamma_H = 1.27 - 0.0067H \quad (22)$$

Age of loading correction factor, γ_{to}

$$\gamma_{to} = 1.25t_0^{-0.118} \quad (23)$$

for moist curing, and

$$\gamma_{to} = 1.13t_0^{-0.094} \quad (24)$$

for steam curing

Slump correction factor, γ_s

$$\gamma_s = 0.82 + 0.00264s \quad (25)$$

where *s* is the slump of fresh concrete (mm). Volume-surface ratio correction factor, γ_{vs}

$$\gamma_{vs} = \frac{2}{3}\left(1+1.13e^{-0.0213\left(\frac{V}{S}\right)}\right) \quad (26)$$

where, V/S is the volume-surface ratio (mm)

Air content correction factor, γ_α

$$\gamma_a = 0.46 + 0.09\alpha \geq 1 \quad (27)$$

where, α is the air content (%).

Fine aggregate correction factor, γ_ψ

$$\gamma_\psi = 0.88 + 0.0024\psi \quad (28)$$

where, ψ is the fine aggregate to total aggregate by weight (%).

Following parameters and coefficients were considered while making calculations for experimental mixes using ACI 209R-92 model

- Type of curing was considered as moist curing.
- Unit weight of concrete was considered as 2400 kg/m^3.
- All the samples were concrete cylinders having diameter 150 mm and height 300 mm.
- Relative humidity of environment during curing and loading was maintained at 60% and same was used for calculations.
- All the other factors were calculated using above mentioned formulas by using different values of f_{cm}, t, t_0, slump, ratio of fine aggregate to total aggregate, air content and other parameters associated to individual mixes.

3.5 Creep as per GL2000 model

This original GL 2000 Model [20] was developed by Gardner and Lockman in 2001. The creep compliance, $J(t, t_o)$ contains two parts: elastic and creep strain.

$$J(t,t_0) = \frac{1}{E_{cmto}} + \frac{\varphi(t,t_0)}{E_{cm28}} \quad (29)$$

$$E_{cmt}(\text{MPa}) = 3500 + 4300f_{cmt}^{0.5} \quad (30)$$

$$f_{\text{cmt}} = f_{\text{cm28}}\beta_e^2 \quad (31)$$

$$\beta_{\text{e}} = e^{\left(\frac{s}{2}\right)\left(1-\left(\frac{28}{t}\right)\right)^{0.5}} \quad (32)$$

where s is CEB style strength development parameter related to cement type.

The correction term for effect of drying before loading $\varphi(t_c)$, could be determined as:

if $t_o = t_c$, $\varphi(t_c) = 1$, if $t_o > t_c$, $\varphi(t_c) = [1-((t_o-t_c)/(t_o-t_c + 0.12(V/S)^2))^{0.5}]^{0.5}$

$$\varphi(t,t_0) = \varphi(t_c)\left[\frac{2(t-t_0)^{0.3}}{(t-t_0)^{0.3}+14} + \left(\frac{7}{t_0}\right)^{0.5}\left(\frac{t-t_0}{t-t_0+7}\right)^{0.5} + 2.5(1-1.086h^2)\left(\frac{t-t_0}{t-t_0+0.12\left(\frac{V}{S}\right)^2}\right)^{0.5}\right] \quad (33)$$

Table 7 – Strength development factor (s) based on type of cement

Cement type	s
I	0.335
II	0.4
III	0.13

Following parameters and coefficients were considered while making calculations for experimental mixes using GL 2000 model

- Strength development parameter (s) related to cement type was taken as 0.13.
- Since all the samples were cylindrical concrete specimen having diameter 150 mm and height 300 mm, V/S was taken as 0.03
- Relative humidity of environment during curing and loading was maintained at 60% and same was used for calculations.
- All the other factors were calculated using above mentioned formulas by using different values of f_{cm}, t, t_0 and other parameters associated to individual mixes.

3.6 Comparison of creep models

There are several differences in the influence factors, formula forms, applicable scope and prediction accuracy of these models due to limitation of specific experimental condition and the emphasis of different researchers. Few common parameters are used by all the five models (B3, FIB model code 2010, AASHTO 2014, ACI 209R-92 and GL 2000 model) discussed in the paper. However, B3 Model consider additional parameters than FIB model code 2010 and same have been listed in Table 8 below.

The magnitude and the rate of development of creep depends upon many factors such as composition of concrete mix, environmental conditions and load level. In terms of applicability, the use of B3 and AASHTO 2014 model is restricted to concrete having 28-day standard cylinder compression strength of 15 to 70 MPa. Similarly, the use of GL 2000 model is restricted to concrete having compressive strength in the range of 16 MPa to 82 MPa. However, FIB model code 2010 is applicable to both normal and high strength concrete up to 130 MPa. Restrictions based on grade of concrete have not been suggested for application of ACI 209R-92 model. Therefore, creep related calculations for high strength concrete using B3, AASHTO 2014 and GL 2000 models may show deviations from the corresponding experimental creep values. Factors and parameters associated with the use of mineral and chemical admixtures in the concrete are not taken into account by any of the above mentioned five models. FIB model adopted new functions and correction factors which modifies long term behaviour of concrete for prediction and for wider applicability.

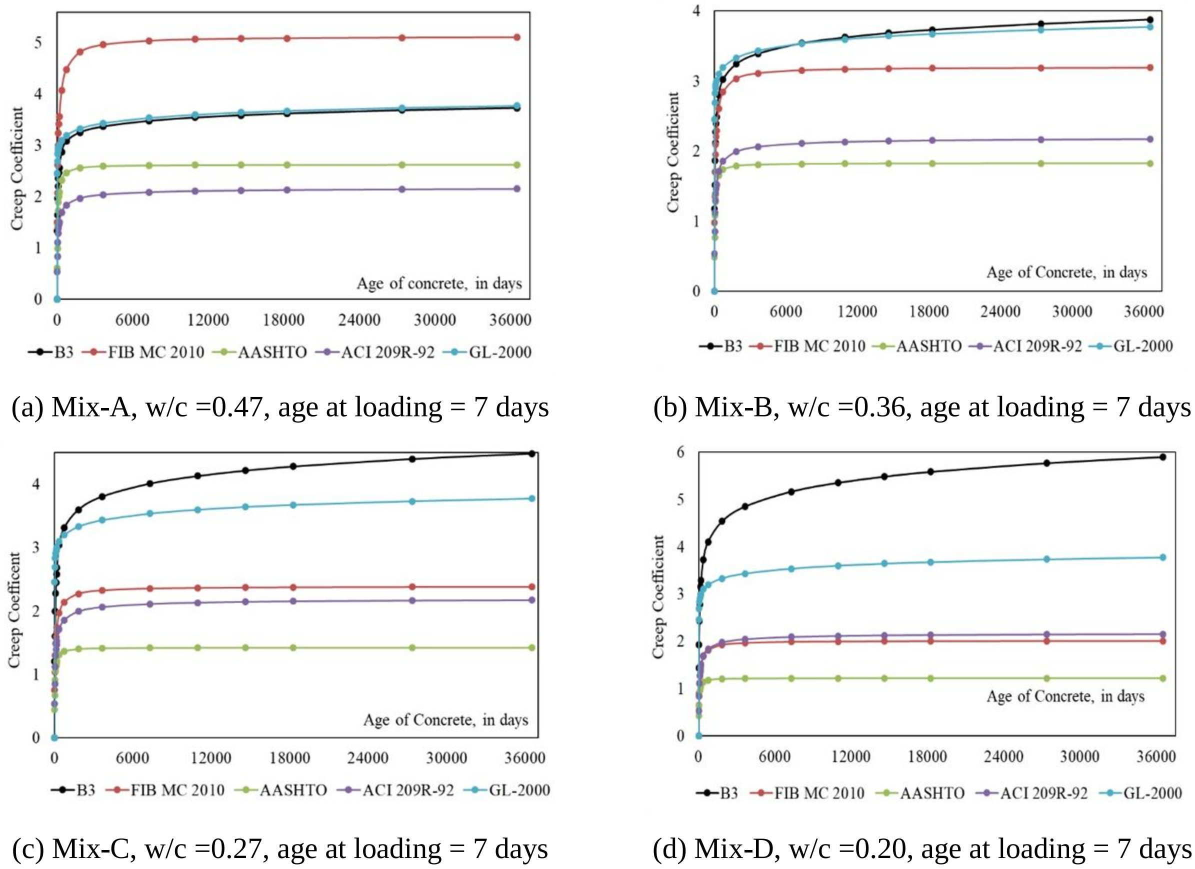
Table 8 – Parameters required by analytical models for prediction of creep

Parameter	**Creep models**				
	B3	**FIB 2010**	**AASHTO 2014**	**ACI 209R-92**	**GL 2000**
Concrete Unit Weight			√	√	
Effective Thickness		√			
Volume-Surface Ratio	√		√		√
Cross Section Shape of Member	√		√		√
Cement Content	√				
Water Content	√				
Water-Cement Ratio	√				
Aggregate-Cement Ratio	√				
Fine Aggregate Percentage				√	
Cement Type	√	√		√	√
Curing Method	√			√	
Slump				√	
Air Content				√	
Relative Humidity	√	√	√	√	√
Age of Concrete at loading	√	√	√	√	√
Age of Concrete at drying (end of curing)	√				√
Compressive Strength at loading					
Compressive Strength at 28 days	√	√	√	√	√
Temperature of curing & environment					
Factors associated with chemical admixture					
Factors associated with mineral admixture					
Aggregate dependent parameter scaling factor					

4 Creep coefficient as per creep models

Creep coefficient of four different mixes (A, B, C & D) as shown in Table- has been worked out using Bazant's B-3, ACI 209-R 92, AASHTO 2014, GL-2000 and FIB model code 2010. The creep coefficients are determined for three different ages at loading (7, 28 and 365 days) for design life of 100 years and relative humidity of 60% (Figure 1 to Figure 3). The comparison of creep coefficients as per different models indicates that there is sharp increase in creep coefficient for each model upto around 365 days age. The rate of increase of each model drastically slows down after 365 days irrespective of the grade of concrete. Both B3 model and GL 2000 shows higher creep coefficients at early age except in case of mix A having water to cementitious ratio of 0.47 and age at loading of 7 days. The AASHTO 2014 Model in general gave the lowest values of creep coefficient except in case of mix A having water to cementitious ratio of 0.47 and age at loading of 7 days. The rate of increase in creep coefficient after 365 days age in case of B3 Model is relatively higher than other models. Both ACI and FIB model code 2010 gave creep coefficients in between the B3 and AASHTO models except in case of mix A having water to cementitious ratio of 0.47 and age at loading of 7 days and similar trend is observed in higher grades of concrete. The magnitude of creep coefficient depends on a wide range of factors including the stress range, element size, concrete mix, coarse gravel content, cement content, type of cement, water/cement ratio, relative humidity, temperature, time of loading, type and duration of curing and maturity. Including most of these factors in creep coefficient calculations is tedious. B3 Model and ACI 209R-92 requires most numbers of parameters for creep prediction. FIB Model code 2010, GL 2000 Model and AASHTO 2014 Model require less number of parameters to predict the creep coefficient.

In order to check the performance of these models for high strength concrete; an experimental study has been conducted with two mixes EM-1 and EM-2 with water to cementitious ratio of 0.47 and 0.20 respectively and results are discussed in paragraph 5.



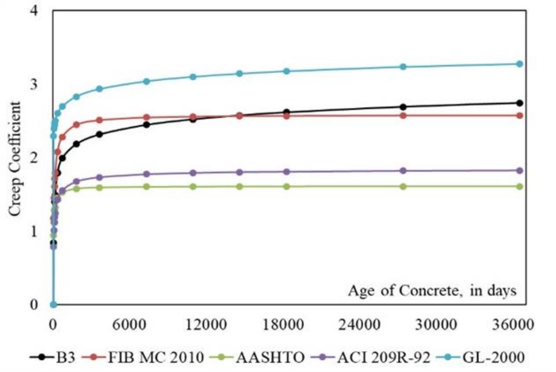
(a) Mix-A, w/c =0.47, age at loading = 7 days

(b) Mix-B, w/c =0.36, age at loading = 7 days

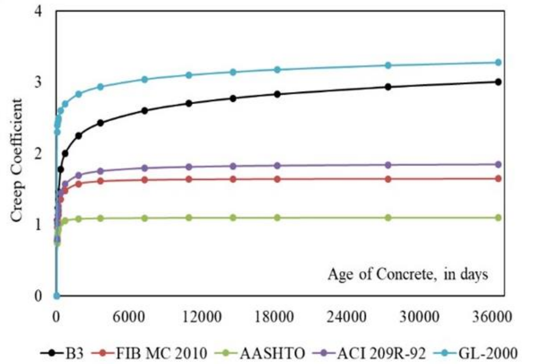
(c) Mix-C, w/c =0.27, age at loading = 7 days

(d) Mix-D, w/c =0.20, age at loading = 7 days

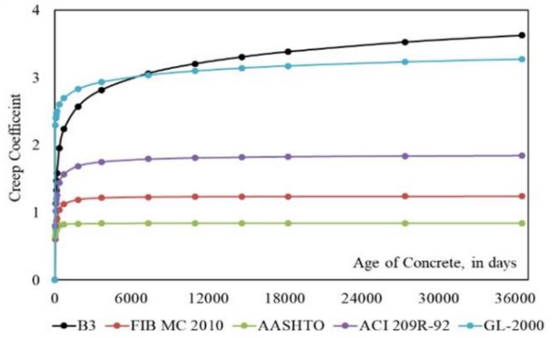
Fig. 1 – Comparison of creep coefficient of concrete mix (a) Mix-A, (b) Mix-B, (c) Mix-C, (d) Mix-D with different creep models (age at loading of 7 days)



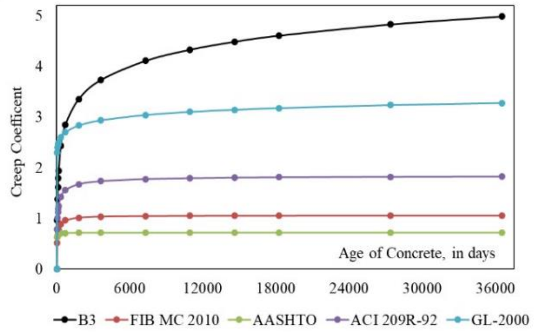
(a) Mix-A, w/c =0.47, age at loading = 28 days



(b) Mix-B, w/c =0.36, age at loading = 28 days

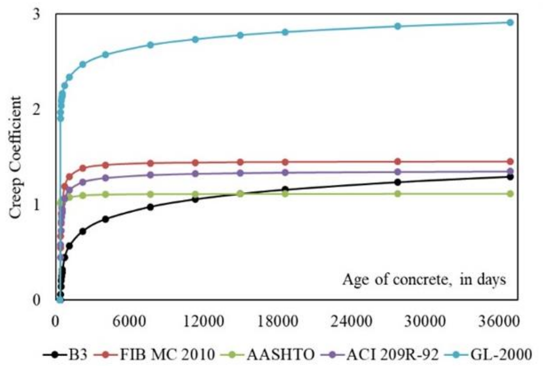


(c) Mix-C, w/c =0.27, age at loading = 28 days

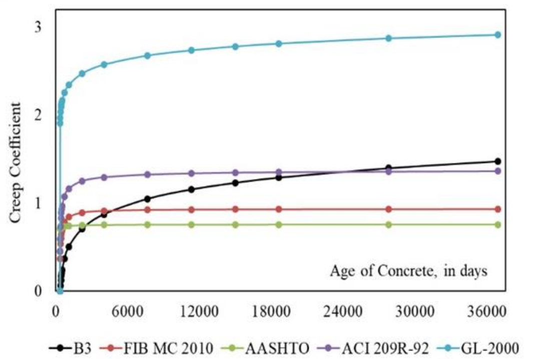


(d) Mix-D, w/c =0.20, age at loading = 28 days

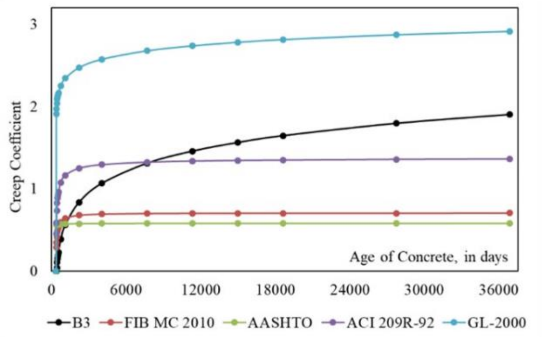
Fig. 2 – Comparison of creep coefficient of concrete mix (a) Mix-A, (b) Mix-B, (c) Mix-C, (d) Mix-D with different creep models (age at loading of 28 days)



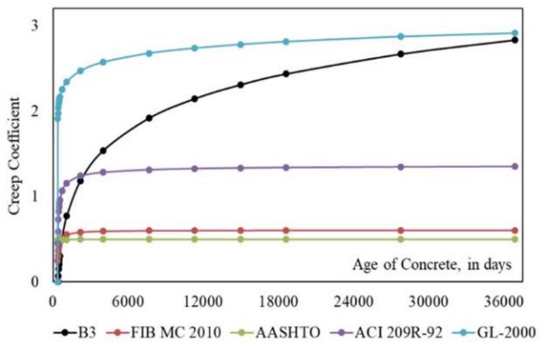
(a) Mix-A, w/c =0.47, age at loading = 365 days



(b) Mix-B, w/c =0.36, age at loading = 365 days



(c) Mix-C, w/c =0.27, age at loading = 365 days



(d) Mix-D, w/c =0.20, age at loading = 365 days

Fig. 3 – Comparison of creep coefficient of concrete mix (a) Mix-A, (b) Mix-B, (c) Mix-C, (d) Mix-D with different creep models (age at loading of 365 days)

5 Experimental creep study in compression

Creep is the continuous increase of the strain in concrete without any change in the applied stress. Creep depends on several factors, including mixture proportioning, environmental conditions, curing conditions, geometry of concrete member, loading history and stress conditions. Creep of concrete depends on the stress in the concrete, age at loading and the duration of loading. As long as the stress in concrete does not exceed about 40 percent of characteristic compressive strength, creep may be assumed to be proportional to the stress. The creep co-efficient $\phi(t, t_o)$ is given by the equation:

$$\phi(t, t_0) = \frac{\varepsilon_{cc}(t)}{\varepsilon_{ci}(t_0)} \qquad (34)$$

where, $\varepsilon_{cc}(t)$ = creep strain at time $t > t_0$, (This does not include the instantaneous strain in concrete at the time of loading), $\varepsilon_{ci}(t_0)$ = initial strain at loading, and t_0 = age of concrete at the time of loading

The creep test was carried out on a cylindrical specimen of size 150 mm diameter and 300 mm height as per ASTM C-512 for concrete with water to cementitious ratio of 0.47 (EM-1) and 0.20 (EM-2) with same mix proportions as shown in table 3 for mix A and mix D respectively. The compressive strength of each mix was used for calculation of the load to be applied to the specimens, which was taken as 40% of the average compressive strength. The cylinders were sulphur capped before being stacked up on top of one another in the creep rig. The vibrating wire strain gauges were inserted in cylindrical specimens at the time of casting. The specimens were cured by wrapping in Butyl Rubber Sheet up to 28 days**.** Relative Humidity was maintained at 60% and temperature was maintained at 27^{0}C. The temperature and relative humidity were maintained at same level after 28 days as well. The creep as per ASTM C-512 is being measured using manual data readout units. In creep test, samples are kept in controlled and loaded condition for the time period of 180 days (Figure 4). Each strength and control specimen was kept under the same curing and storage treatment as the loaded specimen.

The steps for calculating creep strain at a given age are as follows:

EM-1: Water Cementitious Ratio: 0.47and Average: f_{cy}: 45.66 MPa

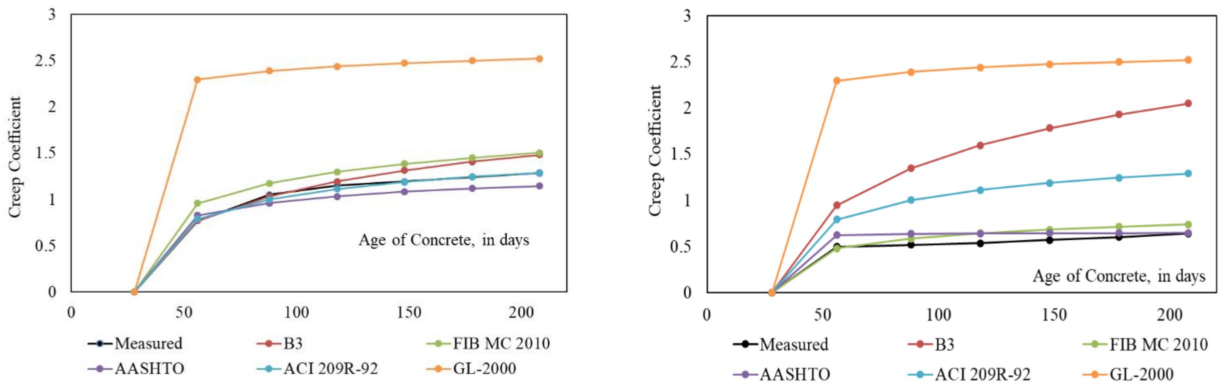
Stress applied: 18.26 MPa (40% of f_{cy})
Total load applied: 323 kN
Age at the time of loading: 28 days
Average strain immediately after loading at time t_0 = 484.31 (μ-strain)
Average strain of unloaded specimens at the time of loading at time t_0 = 19.03 (μ-strain)
Load induced strain per unit stress immediately after loading = (484.31-19.03)/18.26 = 25.48 (μ-strain/ (MPa))
Average strain of loaded specimens at 180 days of loading = 1321.08 (μ-strain)
Average strain of unloaded specimens at 180 days of loading = 258.57 (μ-strain)
Load induced strain per unit stress at 180 days of loading = (1321.08-258.57)/18.26 = 58.19 (μ-strain/ (MPa)
Therefore, the Creep strain per unit stress = (58.19-25.48) = 32.71 μ-strain/ (MPa)

EM-2: Water Cementitious Ratio: 0.20 and Average f_{cy}: 100.21 MPa

Stress applied: 40.08 MPa (40% of f_{cy})
Total load applied: 708 kN
Age at the time of loading: 28 Days
Average strain immediately after loading at time t_0: 1006.80 (μ-strain)
Average strain of unloaded specimens immediately after loading: 0.00 (μ-strain)
Load induced strain per unit stress immediately after loading = (1006.80 – 0)/40.08 = 25.11(μ-strain)
Average strain of loaded specimens at 180 days of loading = 1784.05 (μ-strain)
Average strain of unloaded specimens at 180 days of loading = 131.77 (μ-strain)
Load induced strain per unit stress at 180 days of loading = (1784.08-131.77)/40.08 = 41.22 (μ-strain/ (MPa)
Therefore, the Creep strain per unit stress = (41.22-25.11) = 16.11 μ-strain/ (MPa)

Test results of creep up to 180 days are given in Table 9.

Fig. 4 – Creep testing arrangement



(a) EM-1, w/c =0.47, age at loading = 28 days (b) EM-2, w/c =0.20, age at loading = 28 days

Fig. 5 – Comparison of creep coefficient of concrete mix EM-1 having (a) w/c = 0.47 (b) w/c = 0.20 and age at loading of 28 days with different creep models

Table 9 – Test results of creep up to 180 days with water to cementitious ratio 0.20 and 0.47

Age of concrete (Days)	**Duration of loading (Days)**	**Avg. Total Strain Loaded Samples A (μ-strain)**	**Avg. Strain Unloaded Samples B (μ-strain)**	**Total load induced strain C =A-B (μ-strain)**	**Total Load Induced Strain per unit stress (μ-Strain/ MPa) D**	**Load induced Strain per unit stress Immediately After Loading (μ-Strain/ MPa) E**	**Creep Strain per unit stress (μ-Strain/ MPa) D-E**
Water to Cementitious Ratio 0.20							
28	0	1006.80	0	1006.80	25.11	25.11	0
56	28	1528.45	41.29	1487.16	37.10	25.11	12.49
88	60	1601.53	73.54	1527.99	38.12	25.11	13.01
118	90	1648.27	104.16	1544.11	38.52	25.11	13.42
148	120	1694.81	114.50	1580.31	39.42	25.11	14.31
178	150	1738.40	125.82	1612.58	40.21	25.11	15.10
208	180	1784.05	131.77	1652.28	41.22	25.11	16.11
Water to Cementitious Ratio 0.47							
28	0	484.31	19.03	465.28	25.48	25.48	0
56	28	958.61	135.75	822.86	45.06	25.48	19.58
88	60	1143.93	189.75	954.18	52.25	25.48	26.77
118	90	1200.90	199.82	1001.08	54.82	25.48	29.34
148	120	1230.97	210.38	1020.59	55.89	25.48	30.41
178	150	1273.54	229.91	1043.63	57.15	25.48	31.67
208	180	1321.08	258.57	1062.51	58.19	25.48	32.71

6 Comparison of experimental strains with models

The creep coefficients are determined experimentally for EM-1 and EM-2 for age at loading of 28 days and upto 180 days loading duration and relative humidity of 60% (Figure 5 and Figure 6). The test results of the experimentally obtained creep coefficient values for experimental mixes EM-1 and EM-2 has been compared with Bazant's B3 model ACI 209-R 92, AASHTO 2014, GL-2000 and FIB model code 2010. The results indicate that experimentally obtained creep coefficients for water cementitious ratio of 0.47 (normal strength concrete) are closer to corresponding creep coefficients predicted using all the models except GL2000. However, in case of high strength concrete, B3 model, GL-2000 and ACI 209-R 92 predicts higher values of creep coefficient when compared with experimentally obtained creep coefficients for water cementitious ratio of 0.20. The results indicate that experimentally obtained creep coefficients for high strength concrete are closer to corresponding creep coefficients obtained using FIB model code 2010 and AASHTO 2014 model.

7 Conclusions

Based on the comparison of creep coefficients using Bazant's B-3, ACI 209-R 92, AASHTO 2014, GL-2000 and FIB model code 2010 and the experimentally obtained creep coefficients; following conclusions can be drawn:

(1) The comparison of creep coefficients as per different models indicates that there is sharp increase in creep coefficient for each model upto around 365 days age. The rate of increase of each model drastically slows down after 365 days irrespective of the grade of concrete.

(2) Both B3 model and GL 2000 shows higher creep coefficients at early age except in case of mix A having water to cementitious ratio of 0.47 and age at loading of 7 days. The AASHTO 2014 Model in general gave the lowest values of creep coefficient except in case of mix A having water to cementitious ratio of 0.47 and age at loading of 7 days. The rate of increase in creep coefficient after 365 days age in case of B3 Model is relatively higher than other models. Both ACI and FIB model code 2010 gave creep coefficients in between the B3 and AASHTO models except in case of mix A having water to cementitious ratio of 0.47 and age at loading of 7 days and similar trend is observed in higher grades of concrete.

(3) The results indicate that experimentally obtained creep coefficients for water cementitious ratio of 0.47 (normal strength concrete) are closer to corresponding creep coefficients predicted using all the models except GL2000. However, in case of high strength concrete, B3 model, GL-2000 and ACI 209-R 92 predicts higher values of creep coefficient when compared with experimentally obtained creep coefficients for water cementitious ratio of 0.20. Use of B3, GL 2000 and AASHTO 2014 models are recommended for concrete mixes having compressive strength up to 80 MPa. Therefore, creep related calculations for high strength concrete using B3 and GL 2000 models showed deviations from the corresponding experimental creep values. However, AASHTO 2014 model remain exception in this regard and holds good even in the case of high strength concrete. The results indicate that experimentally obtained creep coefficients for high strength concrete are closer to corresponding creep coefficients obtained using FIB model code 2010 and AASHTO 2014 model.

(4) The comparison of experimental data of creep coefficient with all the five models shows that Bazant's B3 model, GL-2000 and ACI 209-R 92 will not hold good for high strength concrete. FIB model code 2010 and AASHTO 2014 model enables a more accurate analysis for both high and normal strength concrete and better assessment of the creep coefficient of concrete structures at the design stage. In FIB model code 2010 and AASHTO 2014, complexity is significantly reduced and a range of influencing parameters are excluded from the model for simplicity and easy adaptation at the design stage.

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